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Mostafa A .A. Mahmoud
Benha University , Faculty of Science,

Reda Abdel-Rahman
Benha University

Abeer S. Elfeshawey
Benha University, Benha, Egypt

Group solution for an unsteady non-Newtonian Hiemenz flow with variable fluid properties and suction/injection

H. M. El-Hawary[a], Mostafa A. A. Mahmoud[b], Reda G. Abdel-Rahman[b], and Abeer S. Elfeshawey[b]†

[a] Department of Mathematics, Faculty of Science, Assuit University, Assuit 71516, Egypt
[b] Department of Mathematics, Faculty of Science, Benha University, Benha 13518, Egypt

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The theoretic transformation group approach is applied to address the problem of unsteady boundary layer flow of a non-Newtonian fluid near a stagnation point with variable viscosity and thermal conductivity. The application of a two-parameter group method reduces the number of independent variables by two, and consequently the governing partial differential equations with the boundary conditions transformed into a system of ordinary differential equations with the appropriate corresponding conditions. Two systems of ordinary differential equations have been solved numerically using a fourth-order Runge–Kutta algorithm with a shooting technique. The effects of various parameters governing the problem are investigated.

Keywords: non-Newtonian fluid, stagnation point, two-parameter group method, variable viscosity

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1. Introduction

Recently, non-Newtonian fluids have attracted considerable attention in many activities due to their theoretical and technical applications in modern technology, such as petroleum reservoirs, ground-water hydrology, nuclear waste disposal, geothermal energy production, biological fluids transpiration cooling, design of solid, catalytic reactors, and food industry. Schowalter[1] was the one who first studied the boundary-layer flow of a non-Newtonian fluid. Several books excellently summarized the field of non-Newtonian fluids, including Astarita and Marrucci,[2] Schowalter,[3] Crochet *et al.*,[4] and Bird *et al.*[5] These fluids have a common character that the stress–strain relationship is nonlinear.

Several types of non-Newtonian fluids exist, and the most common type is the power law fluid, the shear stress of which is given by $\tau = \mu(\partial u/\partial y)^n$. The problem of laminar flows of power-law non-Newtonian fluids has been studied by several authors. Gupta *et al.*[6] analyzed the problem of steady flow of a non-Newtonian fluid past an infinite porous flat plate subject to suction. Zhang and Wang[7] transformed the magnetohydrodynamic boundary layer system for an electrically conducting power-law fluid with certain boundary conditions into a boundary value problem of a third-order nonlinear ordinary differential equation. They established the uniqueness, existence, and nonexistence of self-similar solutions by a rigorous mathematical analysis method. Olajuwon[8] examined the steady temperature field of a reacting non-Newtonian power-law fluid caused by the exothermic reaction of the fluid molecules as it flows in the presence of the thermal radiation over a flat plate.

The stagnation flows have many applications, such as flows over the tips of rockets, aircrafts, submarines, and oil ships. In addition, the introduction of time as the third independent variable in the unsteady problem increases the complexity of the problem. Much attempt has been made to find analytical and numerical solutions by applying certain special conditions and different mathematical approaches. The problem of unsteady stagnation point flow has been extended in various ways. Soundalgekar *et al.*[9] studied the effect of variable wall temperature on the flow of MHD heat transfer with unsteady stagnation point. Labropulu[10] investigated the unsteady two-dimensional stagnation-point flow of a viscous fluid impinging on an infinite plate in the presence of a transverse magnetic field. Fang *et al.*[11] solved the problem of unsteady boundary layer of incompressible stagnation-point flow with mass transfer using a similarity transformation technique. Xu *et al.*[12] applied the Homotopy analysis method to study the unsteady boundary layer flow of a power-law non-Newtonian fluid near the forward stagnation point.

In all the above-mentioned studies, the physical properties of the fluid were assumed to be constant. Actually, they change significantly with temperature. Many authors have studied the effect of variable properties in the studied fluids.[13–17]

Motivated by all the previous studies, we investigate the effects of variable viscosity and thermal conductivity on an unsteady power-law laminar flow near a time-dependent stagnation point with suction/blowing.

In the present analysis, we perform the two-parameter group transformation to reduce the system of partial differ-

†Corresponding author. E-mail: abeer_elfeshawey@yahoo.com